Problem set-I on Magnetostatics.

Q. A current I flows down a wire of radius a.

(a) If it is uniformly distributed over the surface, what is the current density K?

(b) If it is distributed in such a way that the volume current density is inversely proportional to the distance from the axis, what is J?

Solution:

(a) Since we know that surface current density $\vec{K}$ is given by $\vec{K} \equiv \frac{d\vec{I}}{dl_\perp}$, where $d\vec{I}$ is the current flowing in the ribbon (See fig. I) and $dl_\perp$ is the infinitesimal width running parallel to the flow.

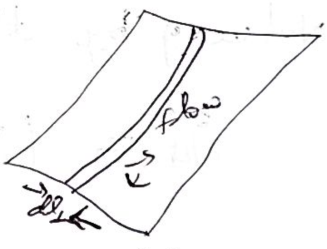

fig. 1

In the above question current I flows down a wire of radius a. The length perpendicular to the flow of the current is the circumference $2\pi a$.

The current density is given by $\boxed{K = \frac{I}{2\pi a}}$

(b) Since $J \propto \frac{1}{r}$; J volume current density, r distance from the axis

or $J = \frac{k}{r}$, where k is a constant ——— (1)

We know that current crossing a surface S is given by (in terms of volume current density J)

$$I = \int_S J\, da_\perp = \int_S \vec{J}\cdot d\vec{a}$$

Now for the given problem

$$I = \int_S J\, da_\perp =$$

$$= \int_S \frac{k}{r}\, r\, dr\, d\phi = k\int_0^a\int_0^{2\pi} dr\, d\phi$$

$$= 2\pi k a$$

$$\Rightarrow \quad k = \frac{I}{2\pi a} \quad \text{--- (2)}$$

From equation (1) and (2) Volume current density J is obtained as

$$J = \frac{I}{2\pi a}\cdot\frac{1}{r}$$

$$\boxed{J = \frac{I}{2\pi a r}}$$